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is grateful to all colleagues
and friends who wanted to be
close to him in this circum-
stance and wishes to add some
clarifying comments on his
opinions

THE PROPER APPROACH TO PROBABILITY

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THE PROPER APPROACH TO PROBABILITY

1. - One, no one, a hundred thousand

"Everybody speaks about Probability, but no one is able to clearly explain to others what a meaning has Probability according to his own conception".

Such remark (of Garret Birkhoff) has been risen rather a long time ago, but such peculiar situation does not appear to be substantially modified till now. Many different and contradictory definitions or pseudo-definitions, of interpretations and ways for applications of probabilistic reasoning have been suggested and used, so as to be led to say,making use of the title of a well known novel by Luigi Pirandello, that they are one, no one, a hundred thousand.

Even worse, most confusion is often arising from mistaking subjective data (like Probabilities and Expectations, concerned always and exclusively with a single event or random quantity), with objective data, like observed successes or failures of single events or observed frequencies, correlations, etc.; many attemps to construct a meningful notion of "objective probability"(seemingly: a mythical idealization of an observed frequency) have been done, but they could not and cannot lead to any outlet, as any strange attempt to construct "a spherical cube".

The only kind of approach which - in my always maintained opinion - leads to remove such ambiguities, is the one where the notion of Probability is accepted - always and exclusively - in his natural meaning of "degree of belief": the one - precisely - of every uncontaminated "man in the street".

Roughly speaking, it may be said that the assertion "the probability P(E) of the event E is (e.g.) 70%" is just the same as saying that one considers as indifferent to have to pay (or to receive) 100 S under the hypothesis that E does occur (and nothing otherwise), or to have to pay (or to receive) 70 S unconditionally. (Many circumstances would usually distort this simplified scheme: the evaluations in terms of utility, the situations of need and particularly urgent need, and so on; but let us pass over such possible complicating factors, which bear no interest for the present exposition).

The presentation chosen and followed in the present short and non technical paper corresponds to the one well understood and accepted as commonsense by any "man in the street", and objectively ascertainable with the approach based on the "proper scoring rules".

The simplest among them is the"Brier's rule" on quadratic basis: if one indicates as p his own evaluation of the probability of a given event E, his penalization is $(E-p)^2$, that is $(1-p)^2$ or $(0-p)^2$ according to the fact that E should occur or should not.

I have been particularly happy of an unexpected confirmation about the correct understanding of probability by laymen. I asked a barman (in Rome) about the meaning of the ternes of numbers (like 50-30-20) indicated - on a table in his glass-window - with reference to each football mach of the next Sunday. "They are the probabilities!" did he exclaim, surprised: no doubt, he wondered to have met the only person in the world ignoring what means "probability". And I was happy, because that was the confirmation of my always maintained opinion that probability is well known and correctly interpreted and applied by everybody, except only the wiseacres aiming to distort the meaning of probability with the idle purpose of transforming it in "something nobler than it is".

But nothing mau become nobler owing to a disguise: at the contrary!

At the contrary, unfortunately enough, most statisticians and philosophers or pseudo-philosophers endeavour to disguise the true nature of Probability with such an impossible aim.

This remark strictly agrees with a french saying (of Poincaré, if I don't mistake):"la théorie des probabilités n'est que le bon sens réduit au calcul".

Forgetting or neglecting such recommandation, or making use of poor "adhockeries" of pretentious complications, one risks to talking nonsense.

2. - Probability and drilling decisions

The best illustration I know, concerned with the proper aid of probabilistic reasoning and of the mathematical developments of the theory of probability with reference to practical problems - chiefly concerned with decision making - may be found (in my opinion, and as far I am informed) in the book by C.Jackson Grayson jr.:"Decisions under Uncertainty: Drilling Decisions by Oil and Gas Operators" (Harvard Business School, Division of Research, 1960).

And I think it would be advisable, when teaching about probability, to make always reference to real problems and situations (like the ones just mentioned) so that any notion and problem and operation implies and suggests the understanding of further problems of interest, more or less connected with the earlier applications.

One of the most remarkable aspects of Grayson's methods is the systematical substitution - with the request of a numerical estimate of the probabilities concerned with a proposed drilling operation - of the more or less ambiguous "evaluative phrases", as e.g., "Control is adequated to recommend", "It contains sufficient merit", "There is a high degree of hazard" (and several more may be found in pp. 54-56).

The answer "probability of success is 70%" conways a better specified information, although only approximately significant and more or less valuable for the Decision Maker according to the confidence in the reliability of the geologist's analysis and conclusions and suggestions.

But this is only the first step. With the same method the whole chain, or tree, of successive decisions may be tested, calculating the expected gains and losses for any one of the possible paths from the starting point the final one (see, e.g., fig. 1)

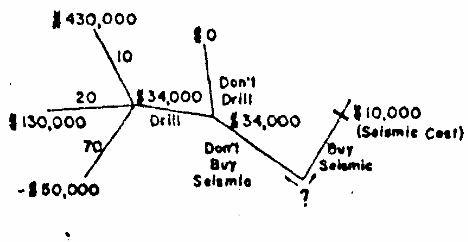

Fig. 1

3. - Probabilistic forecastings on football matches

The most popular kind of forecastings on the result of a football match consists in betting: betting between two people, or bettings systematically organized by authorized institutions as, in Italy, the "Totocalcio".

I completely dislike such blameful kind of betting: it does not, in fact, reward the authors of reasonable previsions, but, just at the contrary, an extraordinarily high reward may be only wen by a foolish indicating so extravagant and surprising forecasts which - if by chance they did occur - are making him the winner of the whole amount allotted to be subdivided among all winners.

"Probabilistic forecastings"is a completely different kind of competition concerned with football forecastings; it is the one experienced during several years at our University (of Rome). The diversity consists in the fact that any participant was required to indicate for every match - instead of the rough distinction of the "results" (1=Win, X=Tie, 2=Lose; always with reference to the home-team) - the probabilities of each one of the three possible results.

A very simple and significant graphical representation shoul clarify, exactly and intuitively, the spirit of such probabilistic competition.

The three vertices of the triangle (see fig. 2) represent the three possible results (always with the reference to the home-team), and are indicated by P_1, P_X, P_2, and the corresponding probabilities by p_1, p_X, p_2 ($p_1 + p_X + p_2 = 1$). As for our application to football forecastings, it is therefore clear that every interior point may represent a particular probabilistic forecast. Moreover, the probabilities p_1, p_X, p_2

are also the distances of $\underline{\underline{P}}$ from the three sides. The moment of inertia (considering the probabilities as masses) is a minimum if we imagine P as the center of rotation.

It is a pity that, in a period of strikes and other circumstances, such experiment had to be interrupted. To begin again is always difficult; but may be some colleagues (perhaps from abroad) could be willing to take it up again; I would be happy if such attempt could be successful, and if the organizers will keep me informed about all interesting remarks on the project.

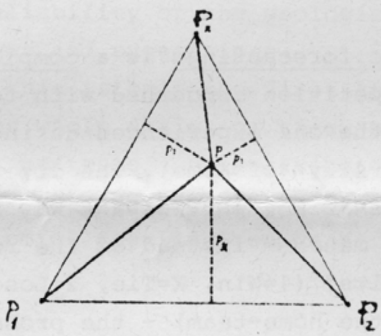

Fig. 2

CONCORSO PRONOSTICI CALCIO 1970-71

24 GIORNATA 18 APRILE 1971

MATR	NOME	CL. GIOR. CL	CL. GIOR. PENAL.	CLASS. GENERALE DA	CLASS. GENERALE A	CLASS. GENERALE PENAL.	SQUADRE MATR	SQUADRE NOME	SQUADRE PENAL.
151	BONI	10	2.180	1	1	52.726			
20	RIZZI	11	2.192	2	2	54.500			
13	VALENTINI	71	2.667	3	3	55.142			
10	DE MARCHIS	70	2.667	4	4	55.321			
15	DE PAOLA	72	2.667	5	5	55.579	1	G.CES.	[illegible]
30	MARINI	12	2.205	6	6	55.532			
155	SIROTTI	26	2.303	8	7	55.930			
11	DE MARCHIS	77	2.757	7	8	56.167			
14	CAMPOLI	79	2.757	9	9	56.480			
31	CARANTI	13	2.205	11	10	56.738			
150	DE FINETTI	19	2.250	13	11	57.046	15	MATEM.	[illegible]
100	MEDIA	27	2.320	14	12	57.181			
[illegible]	[illegible]UCCI	6	2.082	15	13	57.232			

Fig. 3

4. - The need for unequivocal terminologies

Let us mention but a few examples, in order to call attention on this need.

Statisticians use "event" as a collective name whilst the single instances are called "trials" of such "event". But, nevertheless, they speak of the probability of the event as if the inclusion of some "trials" in a collectivity having a common name (e.g., "coin tossings" or "hittings to a target") should imply, ipso facto, that they are "equally likely" because "equally denominated", and also "independent" (if not differently stated); nevertheless they say sometimes that "the probability may be different from a trial to another", and that "the trials may be not independent" so that ambiguities are not easly avoided.

The most significant example for such need of clarifying the language is that of avoiding the contradictory and misleading term of "independent and equally likely events... with unknown probability".

In this case - a proper terminology is necessary in order to avoid misunderstandings! - "independence" does not exist, because the result of any"trial" (to use once the current terminology) is informative, so that it modifies the probability of the future "trials" as evaluated by everybody in such a situation. It is because of the necessary elimination of this ambiguities that I suggested to use a specific denomination instead of the criticized one:

I began earlier to suggest "equivalent" events; later I felt such denomination not sufficiently expressive, and adopted the one of "exchangeable" events; till now I did not find a better one, and probably "Exchangeability" is the best name (so much more that it seems accepted by many colleagues, also from abroad).