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Abstract

In the insurance literature it is often argued that private markets can provide insurance against 'risk' but not against 'uncertainties' in the sense of Knight (1921) or Keynes (1921). This claim is at odds with the standard economic model of risk exchange which, in assuming that decision-makers are always guided by precise point-valued subjective probabilities, predicts that all uncertainties can, in theory, be insured. Supporters of the standard model argue that the insuring of highly idiosyncratic risks by Lloyd's of London proves that this is so even in practice. The purpose of this paper is to show that Bruno de Finetti, widely regarded as one of the three founding fathers of the subjective approach to probability assumed by the standard model, actually made a theoretical case for uncertainty within the subjectivist approach. We draw on empirical evidence from the practice of underwriters to show how this case may help explain the reluctance of insurers to cover highly uncertain contingencies.

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1. Introduction

In a recent article, Kyburg (2002, p. 15) claims that insurance provides 'the simplest and clearest use of probability as a guide in life' and discusses the meaning of probability with respect to the possibility of providing coverage against unique events. He observes that there are insurance companies who issue policies on blatantly "unique" objects or events, and indeed Lloyd's of London is famous for its willingness to insure almost anything for the right premium. Kyburg concludes that since 'from the point of view of the insurance company, every insurance contract is a single case, which is either won or lost…it is curious… that very little of the philosophical literature discusses insurance' (Kyburg 2002, p. 15).

Scholars interested in the philosophical foundations of probability and its application to economic theory have however devoted great attention to the possibility of applying probabilistic reasoning to the insuring of single cases. The preconditions for the insurability of a specific event are central to Knight's (1921) distinction between risk and uncertainty, where uncertainty is specifically defined as a risk that is not insurable. Keynes (1921) tackles the same issue in his *Treatise on Probability* when distinguishing cases in which degrees of beliefs can be represented as numerically definite probabilities and cases in which they cannot.

Nevertheless, the philosophical underpinnings of Knight's and Keynes's views on the distinction between risk and uncertainty have been rendered redundant in the eyes of economists accustomed to the Subjective Expected Utility (SEU) model associated with Savage (1954), which incorporates the 'orthodox' subjective conception of probability widely regarded as stemming from the work of Bruno de Finetti (1937) along with Frank Ramsey (1926) and Savage himself. In terms of the SEU model, the probability of a proposition or event is the strength of the decision-maker's degree of belief in that proposition or event, and it is assumed that the decision-maker always behaves 'as if' she assigned numerical probabilities to the events impinging on her actions, and calculates the value of any risky option as the sum of the probability-weighted utility of its possible consequences. The rationale for adopting the SEU model to analyse the insurance market is straightforward: since the insurer can attach sharp numerical probabilities to every event and thus makes the insurance mechanism work, the standard economic model of risk exchange (the so-called Arrow-Borch model) predicts that all the individual uncertainties will be insured, and that competition in the insurance market will lead to a Pareto-efficient allocation of risk (Gollier 2005, 2007).

The trouble with this view is that the predictions of the economic model of risk exchange are clearly contradicted even by casual observation. To be sure, some idiosyncratic risk associated with specific events may find insurance cover as in the practice of Lloyd's of London (Borch 1976). But insurance markets still fail to insure risks such as terrorist attacks, large environmental catastrophes, transmissions of new diseases, genetic manipulation and so on. And even though the economics literature has provided various explanations of these failures (e.g. asymmetric information, transaction costs and problems of liability),[1] the reports of insurance firms and financial consultants often address this issue in terms of 'unknown probabilities'. That is to say, it is argued in these reports that whenever it is not possible to attach sharp numerical probabilities to certain contingencies, the insurer is not able to calculate an appropriate premium, and thus make the actuarial mechanism work (Swiss RE 2005; Taylor and Shipley 2009).

As the issues involved here concern the theoretical foundations of unknown or absent probabilities, the philosophy of probability is a good place to start (Kyburg 2002). Indeed, outside a strictly subjectivist interpretation of probability it is taken for granted that different philosophical theories of probability may suggest different decision theoretic approaches (Walley 1991; Billot 1992) and contribute to an explanation of the limits to insurability in the private markets (Jeleva and Villeneuve 2004). However, it has remained unnoticed that de Finetti himself dealt with this issue in generally overlooked parts of his work. Our purpose is thus to highlight and examine de Finetti's theoretical case for a distinction between risk and uncertainty within the subjectivist approach, and his arguments to the effect that even the use of subjective probabilities does not always guarantee the completeness of insurance markets.

We start with Knight's and Keynes's theories of probability and uncertainty, and go on to discuss the consequences of their approaches for the practice of the insurance markets. It turns out that although the two theories of probability advanced by Knight and Keynes are usually classified as opposites, it is legitimate to speak about Knightian and Keynesian uncertainty in the same breath and that both provide an explanation of the failure of the insurance market to insure truly 'uncertain' events. We then briefly recall how de Finetti's work is usually invoked to explain why Knight's and Keynes's distinction between risk and

1 For an analysis of these causes see Gollier (2005).

uncertainty is theoretically meaningless and practically irrelevant to an analysis of the functioning of the insurance market.

We then move on to argue that the received view of de Finetti, which is at odds with the practice of most underwriters confronting idiosyncratic risks, does not exhaustively represent de Finetti's own position on the subject. To see this, we analyse two overlooked excerpts from de Finetti's vast contribution to economics and statistics, published in Italian. The first, which appeared in the 1967 *Economia delle Assicurazioni* and has never been translated into English, comments on Knight's distinction between risk and uncertainty. The second, taken from a 1938 review article on the logical approach to probability translated into English only as late as 1985, offers de Finetti's early thoughts on Keynes's notion on non-numerical probability, a theme later assessed in de Finetti and Savage (1962).

It is our contention that these excerpts show an attitude towards the issue of uncertainty and its justification in the theory of probability that does not conform to the traditional interpretation of de Finetti as the champion of a strictly subjective approach. In particular, we argue that de Finetti's discussion of the key elements underpinning Knight's and Keynes's analysis of uncertainty makes a theoretical case for uncertainty within the subjectivist approach and provides solid theoretical ground for understanding the failure of insurance market to cover highly uncertain events. We conclude with some empirical evidence on the reluctance of insurers to cover highly uncertain contingencies (Hogarth and Kunreuther 1985, 1989, 1992; Kunreuther et al. 1995; Cabantous 2007) and some practical implications of our interpretation of de Finetti.

2. Knight and Keynes on the philosophy of unknown probabilities and the Lloyd's of London

Knight (1921) and Keynes (1921) are often cited in discussions of the risk/uncertainty distinction and the limits of insurability. While their philosophical approaches are in many respects quite different, they had remarkably similar views about when it is possible to determine numerically definite probabilities and when it is not. Moreover, both authors examined the practice of the Lloyd's of London to illustrate their views.

2.1. Knight

In his *Risk, Uncertainty and Profit*, Knight (1921) distinguishes between situations in which the decision-maker is guided by 'known chance' (risk) and situations in which she is not (uncertainty). This distinction is based on an analysis of 'probability situations': situations of 'risk' are those in which it is possible to identify classes of more or less homogeneous trials on the basis of which relative frequencies can be determined; situations of 'uncertainty' are ones in which such classes either do not exist or cannot be identified. In the latter case only 'estimates' can be formulated.[2]

It is clear that Knight's distinction between risk and uncertainty presupposes an objective interpretation of probability, as statistical probabilities are mainly seen as a property of the external world. With respect to situations of uncertainty, Knight observes: 'The liability of opinion or estimate to error must be radically distinguished from probability…for there is no possibility of forming in any way groups of instances of sufficient homogeneity to make possible a quantitative determination of true probability' (Knight 1921, p. 231). While estimates are identified as a 'third type of probability judgment', they are not made the subject matter of accurate probabilistic inquiry: indeed it is fair to say that Knight's analysis does not really engage with philosophical issues.

The adoption of an objective conception of probability explains why Knight maintains that private insurance markets fail to cover 'uncertain' contingencies. According to Knight, only 'an uncertainty which can by any method be reduced to an objective, quantitatively determinate probability, can be reduced to complete certainty by grouping cases' (Knight 1921, pp. 231-32). Insurance activity is then 'an illustration of the principle of eliminating uncertainty by dealing with groups of cases instead of individual cases' (Knight 1921, p. 245), and the application of the insurance principle, converting a larger contingent loss into a smaller fixed charge, strictly depends 'upon the measurement of probability on the basis of a fairly accurate grouping into classes' (Knight 1921, p. 246).[3] It is thus impossible to provide

2 As detailed in Runde (1998), Knight's complete taxonomy of probability situations includes three different types: a) classical or a priori probability, i.e. the ideal case in which numerical probabilities can be computed on general principles, namely where they are assigned to equally probable and mutually exclusive possible outcomes; b) statistical probability, i.e. situations in which frequencies may be derived on the basis of an empirical classification of outcomes obtained in classes of more or less homogeneous trials; c) estimates, i.e. situations in which it is not possible to calculate a priori probabilities or where there are insufficient trials 'like' enough to form a reference class of even more or less homogeneous trials on the basis of which frequencies can be determined. Knightian risk includes cases a) and b) while uncertainty is confined to c).

3 On this view market insurance mostly amounts to an application of the law of large numbers. On this point see Section 4.

insurance in situations of uncertainty, since the events to be insured against ‘are far too unique, generally speaking, for any sort of statistical tabulation to have any value for guidance’. As a matter of fact, in most business decisions ‘the conception of an ‘objectively measurable probability or chance is simply inapplicable’ (Knight 1921, p. 231).

On this perspective, life insurance is the branch of insurance in which contingencies are most accurately measurable and apt to a ‘mathematical’ treatment, while in insurance against sickness and accident, like fire insurance, an objective classification of cases is deemed to be impossible because of the practical difficulties in the measurement of ‘real probability’ in a particular case (Knight 1921, p. 246). Knight does not deny that insurance is sometimes offered under these circumstances, but argues that ‘it is notorious that such policies cost much more that they should’ and ‘insurance does not take care of the whole risk’. Nonetheless he seems to be puzzled by the ‘unusual forms of policies issued by some of the Lloyd’s underwriters’ when insuring the loss of ships at sea or the destruction of crops by storm. In extreme cases, like the insurance offered to a business for whatever reason ‘concerned that a royal coronation will take place as scheduled’, Knight (1921, p. 250) concludes that ‘almost pure guesswork’ substitute for ‘“scientific” rate-making’ .

But as the practice of insuring of unique events does not conform to Knight’s requirement of sufficient trials homogeneous enough to form a reference class of which an objective probability can be determined, there is no clear equivalence between his theoretical classification and the variety of actual insurance of business hazards. Knight recognizes this difficulty but argues nonetheless that insurance is offered in these cases possibly on the grounds of a ‘certain vague grouping of cases on the basis of intuition or judgment’ (Knight 1921, p. 250). This tension between theory and practice does not seem to worry Knight, who concludes his analysis with a long discussion of institutional aspects of insurance markets such as the conservative attitude of competent insurers and the offsetting of losses and gains through consolidation.

2.2. Keynes

In his *A Treatise on Probability*, Keynes conceives probability as a logical relation between a set of evidential propositions and a conclusion. If E is a set of evidential premises and H is the conclusion of an argument, then $p = H/E$ is the degree of rational belief that the probability relation between E and H justifies. On this approach, probabilities are epistemic, as they are regarded as a property of the way in which individuals think about the world. If interpreted as degrees of belief, then probabilities are subjective to the extent that information (and reasoning powers) differ between individuals. They are not, however, subjective in the sense that probabilities are independent of the individuals' opinions. Given a set E of evidential premises and a conclusion H, the probability $p = H/E$ is *objective* and corresponds to the degree of belief it is *rational* for an individual to hold. If E makes H certain, then the conclusion follows directly from the premises and $p = 1$; if the relation between H and E is contradictory, then $p = 0$. In the intermediate cases between these two extremes, in which E provides some but not conclusive grounds for believing (or disbelieving) H, then p lies somewhere in the interval [0,1].

Although probabilities are usually regarded as bearing a definite numerical value in the interval [0,1], Keynes rejects the idea that probabilities can always be given a representation by real numbers[4] and argues against the generally accepted opinion that 'a numerical comparison between the degrees of any pair of probabilities is not only conceivable but it is actually within our power' (Keynes 1921, p. 21). Degrees of belief can be measured numerically only when it is possible either to apply the 'Principle of Indifference' or to estimate statistical frequencies.[5] Since the necessary conditions to apply either the principle of indifference or the frequency approach are rare, in many cases 'no exercise of the practical judgment is possible, by which a numerical value can actually be given to the probability…' (Keynes, 1921, p. 29).

Moreover, Keynes (1921, p. 29) insists that some pairs of probability relations may not even be comparable in qualitative terms: 'So far from our being able to measure them, it is not even clear that we are always able to place them in an order of magnitude'. In some cases, it may not be possible 'to say that the degree of our rational belief is either equal to, greater than, or less than the degree of our belief in another'. Probabilities can be compared if they belong to the same 'ordered series', that is, if they 'belong to a single set of magnitude measurable in term of a common unit'. But probabilities are impossible to compare if they belong to two different arguments and one of them is not (weakly) included in the other. To illustrate his point he introduces a diagram in which different probabilistic paths are drawn, all

4 This idea is instead implied by the definition of frequency probability as the ratio of favourable to total number of cases. Subjectivists are committed to it through the Dutch book argument (de Finetti 1937).

5 In terms of the Principle of Indifference, if each of an exhaustive and mutually exclusive list of indivisible hypothesis H_i (i = 1, 2…n) is judged to be equiprobable relative to E, then the probability $p(H_i/E) = 1/n$ for each i. On the frequency view, the probability of an hypothesis H is p if the relative frequency of H in a large number of repeated trials performed under identical conditions tends to p.

starting with 0 and ending with 1. A linear path accounts for the usual probabilistic representation, ranging from impossibility to certainty, but other different non-linear paths between the extremes which do not lie on the straight line can be imagined, representing what Keynes (1921, p. 42) calls a 'non-numerical probability' or a 'numerically undetermined probability'. Only probabilities lying on the same path, or on paths that have points in common, can be compared among themselves, but 'the legitimacy of such comparison must be a matter for special inquiry in each case' (Keynes 1921, p. 40).[6]

It is not too difficult to see the similarities with Knight's distinction between risk and uncertainty. Although Keynes did not draw an explicit distinction between risk and uncertainty in the *Treatise*, he too draws attention to situations that permit the determination of numerically definite probabilities (analogous to Knightian risk) and situations in which only non-numerical representations are available (analogous to Knightian uncertainty).[7] However, unlike Knight, Keynes attempted to provide a mathematical structure for these non-numerical probability values. He in fact tries to give a meaning to a numerical measure of a relation of probability through 'numerical approximation, that is to say, the relating of probabilities, which are not themselves numerical, to probabilities, which are numerical'. Indeed, Keynes (1921, p.76) argues that 'many probabilities, which are incapable of numerical measurement, can be placed nevertheless *between* numerical limits. And by taking particular non-numerical probabilities as standards a great number of numerical comparison or appropriate measurements become possible'. Keynes clearly points to inexact numerical comparison rather than simply to the impossibility of attributing cardinal numbers and deriving probability comparisons (Brady 1993).[8]

Keynes's point is best illustrated by means of one of his own examples. In order to introduce his criticism of the frequentist viewpoint that the numerical character of probability is necessarily involved in the definition of probability as the ratio between favourable cases and the total number of cases, Keynes (1921, pp. 23-32) provides various instances from 'the

6 For a brief, but exhaustive presentation of Keynes's probability theory see Gillies (2000).

7 Keynes's later remarks on uncertainty in the *General Theory* support this viewpoint (Runde 1994b).

8 Keynes's attempt to develop what he called a 'systematic method of approximation' was later taken up by Koopman (1940), who provided an axiomatisation of Keynes's ideas by introducing upper and lower probabilities, thus paving the way for the modern treatment of imprecise probabilities. This literature differs from Keynes in that it does not adopt a logical interpretation of probability, but the mathematical models developing the theme of imprecise, epistemic probabilities have nevertheless drawn heavily from Keynes (Walley 1991). More recently Basili and Zappia (2009) have argued that Keynes's non-numerical probabilities can be interpreted as instances of decision weights that do not meet the standard rule of probabilities and can be represented through a non-additive measure, with the degree of non-additivity representing the degree of distortion of the linear probability prior.

experience of practical men'. Among the concrete cases in which 'no rational basis has been discovered for numerical comparison' he refers to the practice of underwriters. In his examination of 'the willingness of Lloyd's to insure against practically any risk' Keynes rejects the conclusion that this is an argument in favour of the numerical evaluation of all probabilities. Indeed, this practice does not imply that 'underwriters are actually willing … to name a numerical measure in every case, and to back their opinion with money' since it only means 'that many probabilities are greater or less than some numerical measure, not that they themselves are numerically definite' (Keynes 1921, p. 23).

This argument is also reminiscent of Knight. What Knight refers to as an overpriced policy is for Keynes a policy the premium for which exceeds a probable risk that cannot be known. The fact that there is no rational basis for naming a premium attached to an idiosyncratic risk is made clear, in Keynes's view, by observing that different brokers usually offer different premiums even on the basis of the same evidence, and that terms offered on a policy usually vary in reason of the number of applicants.[9] He points out that underwriters themselves distinguish between risks that are properly insurable, either because probabilities are known or because it is possible to make a book that covers all possibilities, and risks that cannot be dealt in the same way and 'cannot form the basis of a regular business of insurance – although an occasional gamble may be indulged with' (Keynes 1921, p. 25). On the basis of his philosophical attitude towards epistemic probabilities, Keynes thus maintains that there may be cases in which probabilities cannot be measured and estimated numerically. As a result, 'the practice of underwriters weakens rather than support the contention that all probabilities can be measured and estimated numerically' (Keynes 1921, p. 25).

3. Insuring unique events: the subjectivist viewpoint as represented by de Finetti

According to de Finetti, the probability of an event or proposition simply represents an individual's subjective degree of belief in that event or proposition. Objective, observer independent probabilities, simply do not exist: 'probability…if regarded as something

9 Discussing the possibility of insuring against the possible introduction of new taxes, Keynes claims that the existence of quotes offered to merchants worried by their effect on business does not imply that the relevant probabilities are known. As a matter of fact, 'that the transaction is in principle one of bookmaking is shown by the fact that, if there is a specially large demand for insurance against one of the possibilities, the rate rises; the probability has not changed, but the "book" is in danger of being upset' (Keynes 1921, p. 24).

endowed with some kind of objective existence, is no less a misleading misconception, an illusory attempt to exteriorize or materialize our true probabilistic beliefs' (de Finetti 1975, p. x).

De Finetti (1937, p. 111) claims that the subjective notion of probability is very close to that of 'the man in the street', that is, the one that is usually applied every day in practical judgments. He argues that a decision-maker's personal belief in a proposition q can be measured by finding the lowest value P_* that she is willing to pay for a gamble that pays S with probability p and T with probability $(1 - p)$.[10] If the decision-maker is an expected value maximizer, she will thus be indifferent between P_* and the gamble: $P_* = p(q)S + [1 - p(q)]\mathrm{T}$. This gives rises to the following real valued probability: $p(q) = (P_* - T)/(S - T)$. In the simple case of playing a gamble that pays S with probability p and nothing with probability $(1 - p)$, $P_* = p(q)S$ and the real valued probability is $p(q) = P_*/S$.

This operational definition of probability makes it always possible to determine the probability of any given event, even if it is unique. And if so, the actuarial mechanism will work in any possible insurance activity. Insurance and gambling may differ in their social and economic functions, but not in their technical features (de Finetti 1957 and 1967).

To see this, consider a situation in which two individuals have to decide whether to exchange a risk. Suppose that an individual A (the insured) is faced with the possibility of a loss L this coming year and that the personal probability of suffering this loss is p. A will be willing to buy insurance as long as the insurance premium she has to pay makes her better off in terms of expected utility. With an endowment of wealth W, the non-insured individual A has an expected utility $\mathrm{EU}^{A}_{n\text{-}i} = p\mathrm{U}(W - L) + (1 - p)\mathrm{U}(W)$. By paying a premium R^A, the individual can insure herself against the loss and get $\mathrm{EU}^{A}_{i} = \mathrm{U}(W - R^A)$. The insured's reservation price is the value $R^A{*}$ that makes $\mathrm{EU}^{A}_{i} = \mathrm{EU}^{A}_{n\text{-}i}$. If the individual is risk-neutral, then $R^A{*} = pL$. In this set-up it is staightforward to see how other hypotheses about the individual's risk attitude apply: a risk-averse individual has $R^A{*} > pL$, depending on the curvature of her utility function, with $R^A{*} - pL$ reflecting the risk premium, that is, the maximum price above the expected loss that she is willing to pay in order to remove risk by purchasing an insurance policy. Risk seeking would imply $R^A{*} < pL$.

10 Although de Finetti defined subjective probabilities in terms of the rates at which individuals are willing to bet on events, thus admitting the possibility that such betting rates could depend on state-dependent marginal utility for money as well as on beliefs, it is assumed here that P and S are numerically definite utility values rather than amounts of money. This difference does not affect the argument at the level of abstraction pursued here. For a discussion of this issue, see for instance Nau (2001).

The insurer's choice problem is symmetrical with that of the insured. Suppose that, on the basis of the available information, and individual B (the insurer) has the same personal degree of belief p regarding the possibility that the insured will experience a loss L. Her problem then consists in evaluating the convenience of exchanging insurance by comparing the utility provided by her own initial capital K and the utility provided by her capital in the case insurance is offered, the latter depending on future gains or losses. An insurer B with wealth K earns $EU^B_{n\text{-}s} = U(K)$ by not selling insurance. By selling a single policy to A at a premium of R^B she can earn $EU^B_s = pU(K - L + R^B) + (1 - p)U(K + R^B)$. As above, the insurer's reservation price for determining whether to sell the policy will be the value R^{B*} that sets $EU^B_s = EU^B_{n\text{-}s}$. And as with the insured, the insurer's reservation price will be $R^B = pL$ if she is risk-neutral, $R^B > pL$ if risk-averse, and $R^B < pL$ if risk seeking.

Since there are no differences in the evaluation of the probability of loss a gain from exchange in this simplified insurance market can emerge only if risk attitudes differ.[11] The more risk-averse the insurer, the higher the premium; the higher the insurer's wealth, the lower the risk premium. The problem is thus to find an insurance price which make both insurer and insured increase their own utility by exchange insurance.

According to de Finetti (1967), the analysis of betting behaviour based on utility provides a complete explanation of the insurance mechanism: it is the difference between individuals' risk attitudes that explains the mutual convenience of exchanging an insurance contract. This explanation is entirely different from the usual starting point in the insurance literature, that is, the existence of a group of units that are subject to the same peril in respect to which it is possible to talk about the frequency of an event (de Finetti 1967, p. 259). On de Finetti's view, the insurer always evaluates the option of offering insurance by considering an 'isolated event'; a set of insurances is thus nothing but the sum of operations relative to 'isolated events'. The fact that there are numerous events does not modify the profitability of each operation *per se*; it simply increases the total amount of profits. The homogeneity of events is irrelevant and eventually negative since homogeneous events are likely to be correlated; the concept of 'compensation' is thus more likely to work in the case of heterogeneous rather than homogeneous events (de Finetti 1967, p. 28). The comparison between the decision-makers' utilities thus provides a general and coherent criterion to explain the insurance mechanism.

11 Trading among individuals with identical risk attitudes can occur if their degree of risk aversion is declining with wealth (or if risk-seeking is increasing with wealth), if $K>W$.

Along much the same lines, subjectivists such as Borch (1976) claim that from a strictly subjective probability perspective, the insurance activity of the Lloyd's of London proves, rather than disproves, the practical irrelevance of the distinction between risk and uncertainty. Lloyd's willingness to insure unique events such as the possible existence of the Loch Ness Monster has been discussed in a famous paper by Borch (1976). Even people working in the insurance industry, Borch reported, find the activities of Lloyd's of London puzzling. This is the typical case in which the risk is neither 'random in nature' nor is it possible to estimate the relevant probabilities from 'available statistics'. Still 'this question was discussed on a higher level at the international congress of actuaries in 1954. On this occasion learned actuaries presented 20 papers which together included 400 pages laying down different conditions which a risk must meet in order to be insurable. All these sets of conditions make it impossible to insure against the capture of a monster in Loch Ness, but still the insurance was written' (Borch 1976, p. 525).

Moreover, when different decision-makers maintain different degrees of belief regarding the occurrence of a given event, subjectivists argue that this increases the propensity to exchange insurance contracts among individuals.[12] Suppose that two individuals are uncertain about a set of mutually exclusive events. They have same objectives, constraints and information; their objective function is strictly concave and the constraint sets convex, so that, given a certain subjective probability distribution over the outcomes, they arrive at a unique decision. In these circumstances, if the decision-makers do not make the same decision, it means that they have different degree of beliefs over the outcomes, and they will be thus willing to make bets with each other about the outcomes (Bewley 1986; Runde 2001; Rigotti and Shannon 2005). It is clear that this phenomenon should have a positive consequence on the insurance market: given the possible difference between the decision-makers' degree of beliefs, people's propensity to exchange insurance contracts should be higher than in a standard model with common beliefs.

To summarize, on the subjective view, the case of 'unknown probabilities' is simply meaningless and it is always possible to attach sharp numerical probabilities to uncertain events and thus make the actuarial mechanism work. The subjective expected utility model provides a simple explanation of the insurance mechanism without appealing to the idea of

12 De Finetti did not go very far in this direction since he claims that the insured does not go into this kind of calculus when she decides whether to pay a premium. The insured does not have the information or the skills to evaluate if the premium is fair or not; she only has the choice to pay or not to pay it (de Finetti 1967, p. 302).

pooling homogeneous events. A portfolio can be seen as a sum of independent single operations; the more heterogeneous the portfolio, the less correlation between the events there is going to be, the easier the ‘compensation’ principle will apply. Moreover, the propensity to exchange insurance contracts comes not only from individual’s different attitudes toward risk but also from the divergences among the decision-makers’ degree of beliefs, which make the betting activity mutually convenient.

4. The “philosophy” of practioners

As seen in the previous section, lying at the core of the insurance mechanism is the idea that the insurer is able to identify and quantify the risk when providing different levels of cover. If she is able to estimate the magnitude of the loss L and the probability p of the loss occurring, then she will be able to determine what premium to charge. If in contrast the probability of the loss is unknown, the insurer is not able to calculate the expected loss, set up an appropriate premium and thus to make the actuarial mechanism work. Notwithstanding the claim of subjectivist interpreters of probability, and the pervasiveness of the maximization of subjective expected utility as criterion for choice in decision theory, there remains a substantial literature claiming that insurance can cope with risk but not with uncertainty.

For instance, in his analysis of social insurance, Atkinson (1995, p. 210) observes that in dealing ‘with actuarial risk, rather than what is called *uncertainty* in the sense of Knight (1921)’, what is missing is ‘the important function of social insurance in providing for contingencies which are not foreseen, or to allay fears about events which we cannot forecast’. Barr (2001a) explains that the problem of unknown probabilities can arise when the insured event is rare, unpredictable because of complexity, or has a long time horizon. He reviews a number of cases in which the notion of unknown probabilities seems to be relevant in explaining the failure of the private market. In particular, he argues that the case of unknown probabilities can explain why the private markets fail to provide a number of insurance services like medical insurance concerning a specific illness (e.g. the extent of risk from exposure to ‘mad cow’ disease); long-term care insurance (due to increase in longevity the probability distribution of care for future cohorts of the elderly is to change over the course of contracts with long-term horizons); macroeconomic shocks; private unemployment insurance concerning a particular individual becoming unemployed; and so on.[13]

Similar views emerge in studies by insurance firms themselves, analyzing the technical conditions to be satisfied for insurers to offer cover against uncertain events. Although it is difficult to create a definitive checklist to distinguish between insurable and uninsurable events, industry participants have proposed several guidelines. For instance, in a recent publication (2005), Swiss Re's risk team, drawing upon Berliner (1982), lists the following criteria of insurability (Tab. 1):

Table 1.

N.	Category	Criterion	Characteristic
1	Actuarial	Risk/uncertainty	Measurable
2		Loss occurrences	Independent
3		Maximum loss	Manageable
4		Average loss	Moderate
5		Loss frequency	High
6		Moral hazard, adverse selection	Not excessive
7	Market-determined	Insurance premium	Adequate, affordable
8		Insurance cover limits	Acceptable
9		Industry capacity	Sufficient
10	Societal	Public policy	Consistent with cover
11		Legal system	Permits the cover

Source: Swiss Re (2005)

This scheme comprises six actuarial criteria, three conditions that reflect the state of insurance market, and two criteria concerning societal factors. Consider the actuarial criteria numbered 1, 2 and 5. The first states that, in order to be insurable, a risk must be measurable in the sense that its likelihood must be known. If the probability of experiencing a loss is not measurable, the insurer cannot set up the insurance premium and make the actuarial mechanism work. The second states that since the insurance mechanism requires a certain number of losers and winners, probabilities have to be independent, that is, it is not possible to insure against so-called 'common shocks'. Risks in the portfolio therefore cannot be overly

13 On this view, social insurance arises as a response to private market failures (Arrow 1963). Although social insurance is modelled on private insurance, since the benefits are related to the contributions record and the occurrence of a specific event, it differs from the private insurance in two important respects: (1) the membership is compulsory and (2) the contract is less specific. These distinctions make it possible to break the link between premium and individual risk – thus departing from an actuarially fair individual insurance – and to cover risks that can change over time (Barr 2001a; Atkinson 1993). This is why 'social insurance, in sharp contrast with actuarial insurance, can cover not only *risk* but also *uncertainty*' (Barr 2001b, p. xxiv). On this point see Feduzi and Runde (2009).

correlated with one another. Finally, the fifth criterion states that, in order to make performance more predictable and less variable, high frequency is needed to apply the Law of Large Numbers (Swiss Re 2005, p. 5).

It is apparent that the meaning of these criteria and their relevance for the question of the limits of insurability depend on the interpretation of probability one adopts. But it is Knight's analysis, and his distinction between risk and uncertainty that is, explicitly or implicitly, adopted by much of the literature on insurance and risk management. On this view, Swiss Re's first actuarial criterion is normally satisfied by adopting a frequentist approach to probability. It is argued that, since the observed frequency of an event approaches the underlying probability of the population as the number of trials increases, it is possible to obtain a notion of underlying probability by observing events that have occurred in the past. By looking at the proportion that a particular outcome has occurred over a long period, under *more or less* the same conditions, it is possible to determine an index of the relative frequency of that outcome; this index represents the probability distribution attached to the outcome, that is, the average rate at which the outcome is expected to occur.

The Law of Large Number (the fifth criterion) provides the basis for calculating these kind of probabilities and is fundamental to the insurance mechanism for two reasons: first, in estimating the underlying probability the insurance company must have a sufficiently large sample; second, once an estimate of the probability has been made, it has to be applied to a large number of units to permit the underlying probability to work itself out. Even if the probability of an event were accurately known, the statistics would not apply to an individual exposure or even a small group (Outreville 1998).[14]

It is thus clear how the Law of Large Numbers (and the related central theorem) is used in the literature on insurance to explain pooling of losses as an insurance mechanism: the average of a large number of independent and identically-distributed realisations of a random variables tends to fall close to the expected value. The entry of additional exposure units to an insured pool tends to reduce the variation of the average loss per policyholder around the expected value. The risk faced by the insurance company is not just the sum of the individual risks, since by grouping exposure units, the company can predict within narrow limits the amount of losses that will actually occur. Risk reduction through pooling, however, is

14 It is claimed that what makes insurance feasible 'is the pooling of many loss exposures, homogeneous and independents, into classes (classes of business), according to the theory of probabilities (the law of large numbers)' (Outreville 1998, p. 132).

substantial only when losses are independent, that is, uncorrelated (the second criterion). It is in fact possible to demonstrate that the magnitude of risk reduction significantly decreases in the case of pooling arrangements with correlated losses (Harrington and Niehaus 1999).

In summary, an insurable risk requires a large group of roughly similar, but not necessarily identical, independent exposure units that are subject to the same peril. In this way, the insurer can predict loss based on the Law of Large Numbers and set up a premium that is sufficient to pay all claims and expenses and guarantee a profit during the policy period. In the presence of 'uncertainty', the chance of loss is unknown, and the insurer is not able to calculate the premium and thus to make the actuarial mechanism work. The market failure is not due to the problem of asymmetric information, but to the impossibility, theoretical and practical, of establishing the probability of the loss.

That de Finetti's strictly subjective perspective is at odds with the practice of most underwriters confronting idiosyncratic risks has been recently reaffirmed under the headings of 'misapplication of probability and statistics in real-life uncertainty'. The current financial crisis has rejuvenated in the press the fancy notions of 'black swans' or 'unknown unknowns', but these terms, referring to outcomes that were not listed in the space of outcomes of the model, in insurance are 'part and parcel of the problem', underwriters claim: in most cases, 'even the subjective workarounds of Bayesianism fail to cover up the cracks' (Taylor and Shipley 2009, p. 1). Taylor and Shipley, who both spent many years in the Lloyd's insurance market, confess that what they usually do is to stand back from their model and take a view as to 'what degree of accuracy it is capable of'. It is their expertise that dictates when or when not to be confident in their models, in order to have a better chance of incorporate unexpected events into their model. Moreover, even when applying Bayesian updating to adjust for evidence there remains the problem of how to choose a prior probability distribution: 'what if we just don't have any grounds for a quantitative probabilistic model at all?'. They conclude that 'in real life we use qualitative methods for decision under high uncertainty and only use probability "numbers" as an illustration' (Taylor and Shipley 2009, p. 3).

Section 6 will show that this practical attitude towards probability assessments, though at odds with a strictly subjectivist viewpoint, has been made the subject of thorough examination in empirical studies under the headings of ambiguity aversion. But let us first consider de Finetti and his usually disregarded "second thoughts" on the significance of Knight's distinction.

5. De Finetti on uncertainty in Knight and Keynes and on insurability

This section is devoted to an exegesis of two excerpts from de Finetti's vast contribution to economics and statistics published in Italian. The first, never translated into English, comments on Knight's distinction between risk and uncertainty. The second, taken from a 1938 review article on the logical approach to probability translated into English only as late as 1985, offers de Finetti's early thoughts on Keynes's notion on non-numerical probability. It is our contention that these two excerpts show an attitude towards the issue of uncertainty and its justification in the theory of probability that does not conform to the traditional interpretation of de Finetti as the champion of a strictly subjective approach. De Finetti had a subjective, but pluralistic view towards the foundations of probability, one that finds evidence also in his major volume on probability, *The Theory of Probability.* Here, drawing on de Finetti and Savage (1962), he discussed the question of whether imprecise probabilities exist and admitted that in certain occasions a straightforward introduction of numerical values for probabilities is not obvious and 'it seems preferable to start from a purely ordinal relation – i.e. a qualitative one – which either replaces the quantitative notion … or is used as a first step towards its definition' (de Finetti 1975, vol. 2, p. 363).[15]

5.1. De Finetti on Knight

In his 1967 *Economia delle Assicurazioni*, de Finetti discusses the relevance of Knight's distinction between risk and uncertainty, and its consequences for the functioning of the insurance market.[16] First, de Finetti points out that Knight's distinction is simply 'implicit' or even 'presupposed' by the frequentist approach. He summarizes the distinction as follows: situations of 'risk' are ones in which uncertain factors can be eliminated (converted into fixed costs) by the concept of 'compensation' or the possibility of transferring them to an insurer who applies the concept of 'compensation' on a large scale;[17] situations of 'uncertainty' are, in contrast, ones in which uncertain factors are not compensable or insurable, and where every

15 To the best of our knowledge this pluralistic attidude has been recognized only in Suppes and Zanotti (1989, p. 23).

16 The volume we are referring to is co-authored with Filippo Emanuelli. However it is specified in its introduction that de Finetti is uniquely responsible for the draft of Part I, while Emanuelli is responsible for Part II. As we quote only from Part I of *Economia delle Assicurazioni* we shall refer to it as de Finetti 1967.

17 By the concept of 'compensation', de Finetti means the tendency to balance out gains and losses over a large number of actions with fair and uncorrelated random outcomes, which is exactly the property of frequencies that tend to approach to probabilities, i.e. tendency towards averaging for positive and negative deviations (de Finetti 1967, p. 29).

individual thus has to deal with these factors alone and on the basis of her own judgment (de Finetti 1967, p. 33). Second, de Finetti observes that Knight's distinction is not linked to the actual possibility of transferring an individual risk to an insurance company, something which depends on institutional and contingent factors, and thus has no interesting conceptual and general meaning. As for the theoretical issue, de Finetti reaffirms that the distinction cannot concern the particular features of a risk which might make it theoretically uninsurable, if this consideration is independent of the fact that the private market actually insures that risk (de Finetti 1967, p. 33-34). Indeed, de Finetti contends that every risk can be insured as long as there is someone willing to accept it, and, unsurprisingly, he argues that the practice of the Lloyd's of London testifies to this (de Finetti 1967, p. 34).

De Finetti suggests that the right way to answer the question of whether Knight's distinction makes any sense is to analyse its relevance within the field of decision theory under uncertainty. The distinction between risk and uncertainty from the individual's standpoint and the distinction between insurable and uninsurable risks from the insurer's standpoint both depend on the same kind of considerations: whether it is always possible to apply probabilistic reasoning when making decisions, or whether it is necessary to distinguish between choice situations in which probabilistic reasoning can be applied and choice situations in which it cannot. De Finetti's answer to this question is straightforward: at a theoretical level, it is always possible to apply probabilistic reasoning when making decisions.[18] This is why he claims that Knight's distinction is theoretically meaningless (de Finetti 1967, p. 35).

Since Knight's distinction ultimately relies on the idea of the existence of objective probabilities, it is not surprising that de Finetti should reject it. Nevertheless, de Finetti ends his analysis with an observation that hints at a different interpretation of Knight's distinction. Specifically, he suggests that 'from this [subjectivist] perspective, what Knight would refer to as "risks" are cases in which one finds minor discrepancies in valuations made by different individuals, or by different insurers. This is what renders them insurable' (de Finetti 1967, p. 36).[19] He continues by arguing that: 'the individual appreciation of the various risks translates (more or less explicitly) into a subjective valuation of the probability which, depending on

18 Applying probabilistic reasoning when making decisions under uncertainty cannot but be a criterion of general validity because it is deduced from simple rules of logical coherence rather than a specific empirical hypothesis (de Finetti 1967, p. 35).

19 This and the following quotations from de Finetti's 1967 volume are translated from the original Italian by the authors.

whether the conditions are favourable, will be roughly uniform amongst the various individuals. This could even lead to the creation of an insurance market in which the valuations that are more or less accepted, constitute the foundation for the setting of premiums' (de Finetti 1967, p. 37). De Finetti maintains that uniformity of judgments is likely to occur in games of chance (whenever individuals calculate probabilities in highly symmetric situations, such as tossing a coin, rolling a die or playing cards) or in respect of events for which statistical historical data is available (for example, accidents, fires or deaths, where individuals' judgments are based on the frequency observed). Outside these cases, in his view, the difference among individuals' subjective probabilities, the degree of which will depend on the particular circumstances under which they are elicited, render certain risks uninsurable (de Finetti 1967, p. 36). De Finetti regards it as a mistake to think that it is possible to clearly distinguish between cases in which perfect uniformity among judgments exists and cases in which it does not (or between cases in which some objective conditions which lead to perfect uniformity among judgments exist and cases in which they do not). As it is always a matter of degree, Knight's main mistake was thus to draw a too sharp distinction between 'risk' and 'uncertainty', since this gives the impression that the distinction is clear-cut and fundamental rather than fuzzy and secondary. It is for this reason that de Finetti (1967, p. 37) ultimately refuses to adopt that terminology.

At least two considerations follow from de Finetti's discussion of Knight's work. First, although de Finetti rejected Knight's distinction between 'risk' and 'uncertainty', he nevertheless hinted at an interpretation that is relevant insofar as it has recently been revived within the subjectivist approach. Following his line of reasoning, a situation of 'uncertainty' may be interpreted as one in which individuals' opinions about the probability of the occurrence of a given event sensibly differ. It is therefore possible to speak of de Finetti's notion of Knightian uncertainty. Second, even though de Finetti regards people as always attaching sharp numerical probabilities to events, he admits that insurance markets may nevertheless fail. It is worth noting that he was a practitioner (he worked at *Assicurazioni Generali* for 15 years) and it is not surprising that he saw that the adoption of subjectivist probabilities does not guarantee complete markets as postulated by the standard economic model of risk exchanges.

Consider the following simple example. Let us say that a number of individuals would like to buy insurance against the occurrence of a given loss L and that the losses are uncorrelated across individuals. Suppose that due to the absence of reliable historical data the insurer has only a rough idea of the probability p that the loss L will occur. Let us say that the insurer asks a number of experts to evaluate the probability of the occurrence of the loss L and that some of the experts estimate that the probability of the loss is p^* while the others estimate that it is p_*, where $p^* > p_*$. The question is whether the insurer should take into account this source of Knightian uncertainty in the process of deciding the premium.

It might be argued that the answer to this question is negative, if it is claimed that Knightian uncertainty is irrelevant because the insurer will in any event always use point-value subjective probabilities to calculate the expected value of the loss.[20] We reject this view, however, since the insurer will still have to find a decision rule to aggregate the experts' opinions. And it turns out that Knightian uncertainty does have bite here. To see why, suppose that the insurer decides that the best that she can do in both cases is to focus on p^* and p_* alone and, on the grounds that she sees no reason to prefer one to the other, to proceed on the basis that they are equally possible. In this case the insurance premium would be based on the mean of the two possible probabilities, that is, $p = (p^* + p_*)/2$, and the expected loss will then be $E(L) = pL$. The problem with this decision rule, however, is that the insurer knows that the average loss is likely to differ from $E(L) = pL$, being either closer to $E(L)^* = p^*L$ or to $E(L)_* = p_*L$. This phenomenon implies that the insurer will perceive a large variance in the distribution of average loss around the expected value $E(L) = pL$.

In the literature on insurance and risk management, this phenomenon is usually referred to as 'exposures with parameter uncertainty', where 'parameter uncertainty causes the distribution of average losses around the insurer's estimate of expected loss per policy holder to have greater variance, which is the same effect as having high correlation in losses' (Harrington and Niehaus 1999, 166). Since the insurer's estimate might be wrong, any error in the estimate of expected loss will apply to all policyholders; in this respect, it is possible to speak in terms of correlation in the insurer's estimation errors across policyholders. In these circumstances, the insurer, in order to reduce the probability of insolvency, is likely to highly overload the insurance premium.

20 This would be the suggestion of scholars such as Savage who, after having recognized that 'there seem to be probability relations about which we feel relatively "sure" as compared with others' (Savage 1954, p. 57), pointed out on normative grounds: 'Some people see the vagueness phenomenon as an objection; I see it as a truth, sometimes unpleasant but not curable by a new theory' (Savage 1962, p. 163).

5.2. De Finetti on Keynes

In his 1938 Italian review of the works of ‘Cambridge probability theorists’, Keynes’s *Treatise on Probability* and Jeffreys’s *Scientific Discovery*, de Finetti praises the renewed interest in the epistemic perspective of scholars engaged in foundational studies. The differences between the objective perspective implicit in the logic of probability endorsed by Keynes and his own subjective interpretation are pointed out (de Finetti 1938, pp. 83-84).[21] But de Finetti supports what he sees as a revival of an epistemic approach to probability blurred by the empiricist perspective of frequency probability, and praises Keynes’s idea of interpreting probability theory as the logic of thinking determining the ‘degree of uncertainty [of propositions] at a given time when there is not enough information to judge them true or false’.[22]

This favourable attitude towards Keynes is not limited to the link between his probability theory and the epistemic approach, and has a counterpart in a specific comment de Finetti makes on the question of measurable probabilities. De Finetti recalls that, when discussing how the theory of probability translate into probability calculus, Keynes admits neither the postulate that each probability is a number between 0 and 1, nor that two probabilities are always comparable one with the other. From de Finetti’s perspective, ‘Keynes’s position is certainly not suited to the development of a mathematical probability theory and is also hardly in keeping with the intuitive idea of probability’ (de Finetti 1938, p. 88). But he also concedes that Keynes’s position deserves consideration with respect to one specific aspect: ‘without denying that for each individual the probabilities for two events must be comparable, it may be that, based on certain assumptions shared by all, certain inequalities already have a determinate sense which is common to everyone's opinion, whereas others vary from individual to individual’. As an example de Finetti (1938, p. 88) adds: ‘one can assume, for example, the equal probability of certain events which are in a certain sense symmetrical, e.g. of a slightly oblong die, one may say that two square faces are equally probable and also that the four oblong faces are equally probable but more probable than the square sides. In that case, we must admit that this probability will fall between 1/6 and 1/4 (and $4a+2b=1$ with $0<b<a$), but it is not determined which values between 1/6 and 1/4 will

21 On the significance of de Finetti’s critique of logical probability see Galavotti (1989) and Gillies (2000).

22 As noted by Gillies and Ietto-Gillies (1987), in this article de Finetti stressed the similarities between his view and that of Keynes. The difference with Ramsey’s comments on the *Treatise* is striking. Ramsey (1926), the other main early proponent of the subjectivist approach, was harshly critical of the logical approach. On Ramsey’s critique and Keynes’s reaction to it, see Runde 1994a.

obtain and, based on the only assumption made, we may expect that each individual will evaluate that probability differently'. De Finetti concludes that, though at odds with his own subjective viewpoint of probability 'it would not be at all irrational to interpret this in agreement with Keynes as an absence of comparability'.

This concession suggests some sympathy with Keynes's insistence on the possibility of non-numerical probabilities and is of relevance for our reconstruction of de Finetti's nuanced understanding of uncertainty. Indeed, following our assessment of Keynes in Section 2 above, non-numerical probabilities may be related to the problem of interval probability or multiple priors, a point de Finetti himself was aware of and discussed in a joint work with Savage, published in Italian in 1962, and never translated into English. This paper, devoted to a discussion of 'how to choose initial probabilities' from a subjectivist perspective, considers the question of whether 'inexactly determined' and 'fuzzy' initial opinions can be expressed through an exact probability value as a secondary issue, but examines it in detail. Here it is conceded that 'it is often practically impossible to anyone to state that … the probability which he can attribute to a certain event has a precise value' (de Finetti and Savage 1962, p. 95), and that imprecision can constitute an 'actual epistemic state' of the individual facing uncertainty whose nature is 'difficult to be made precise in a convincing manner' (de Finetti and Savage 1962, p. 134).

The 1962 paper discusses Smith (1961), who followed Ramsey and de Finetti himself in measuring beliefs by means of betting quotients, but showed that a person consistently rejecting to bet on either an event or its complement can be attributed an interval of initial probabilities.[23] De Finetti and Savage admit that Smith provided a precise criterion to determine two initial probability values p_* and p^* , where $p^* > p_*$, and take the 'fact' that there maybe a 'non-betting zone' for granted. Among the reasons justifying the reluctance of actual individuals to bet on certain events, it is mentioned the case of an insurance firm that specialises in certain insurance fields and reject to insure others. Also it is envisaged that an individual may reject to bet on fields in which she feels herself incompetent (de Finetti and Savage 1962, pp. 136-139). As for the normative content of Smith's interval probabilities, Smith's approach is criticised, but considered of help in those cases in which one has 'partial

23 Smith (1961) presented his work as a generalisation of the subjective approach to admit imprecision. He introduced the fundamental principles of avoiding sure loss and coherence in the context of interval probabilities of Koopman and Good, and interpreted upper and lower probabilities as personal betting quotes. Smith provided also an extension of interval probabilities to statistical inference and decision making, showing that coherent lower probabilities can be seen as lower envelopes of precise probability measures. For an analysis of Smith's role in the development of imprecise probability see Walley (1991).

knowledge' of a preference and the probability of an event can be said to be indeterminate. Among these cases two are singled out as more relevant: first, the case of a number of decision-makers who are to make a collective decision, and, second, the case of a single individual who experiments 'kind of personality dissociation' (de Finetti and Savage 1962, p. 142). The first case in which Smith's approach is admitted to be sound even on normative grounds mostly replicate, indeed prelude to, de Finetti's justification of Knight's notion of uncertainty. The second case coincides with de Finetti's understanding of Keynes non-numerical probabilities.[24]

It is apparent then that de Finetti and Savage's analysis of interval probabilities has clear links with the two excerpts discussing Knight and Keynes, and that it can be related to the modern treatment of imprecision in decision-making. On the multiple prior approach (Levi 1974; Gärdenfors and Sahlin 1982) an event is 'risky', when the decision-makers' beliefs can be represented by a unique single-point probability distribution, and 'uncertain', when the decision-maker's beliefs are represented by a multiplicity of probability distributions. The standard expected utility framework is then modified by replacing the unique subjective probability distribution used in SEU with a set of probability distributions. The multiplicity of the subjective distributions represents the decision-maker's ignorance about the true probability distribution (Bewley 1989).

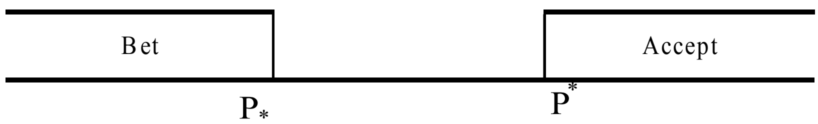
Following Bewley (1986) let us analyse the consequences of the multiple prior approach by examining again the decision-maker's behaviour in betting situations. As discussed, if the decision-maker is willing to exchange the amount P_* for a gamble that pays S if h is true and nothing if h is false, then $p = P_* / S$. Since the subjective approach implies that the decision-maker must be willing to bet or accept bets at the odds that represent her degree of belief (Kyburg 1978), then the decision-maker will be willing to accept an amount P^* in exchange for a gamble that involves a loss of S if h is true and nothing otherwise, such that $P_* / S = -P^* / -S = p(q)$.

Let us now suppose that the decision-maker's beliefs are more vague than the subjectivist approach implies. Suppose, for instance, that the decision-maker has only a rough idea of the value P and, in particular, thinks that P is included in the interval $[P_*, P^*]$, with P^*

24 As an attempt at clarification an example is offered, concerning the unknown area of a given scalene triangle: under certain constraints to its sides the triangle can assume different areas, but this does not mean that the triangle has an indeterminate area, it means only that it can assume different values and there is not enough information to identify it (de Finetti and Savage 1962, p. 142). This example replicates the one used to justify Keynes's argument in the 1938 review, we reported in section 2 above.

$> P_*$. In these circumstances, the decision-maker will be willing to give up any amount $P < P_*$ in order to receive S if q is true and nothing otherwise, and to accept any amount $P > P^*$ for having to give up S if q is true and nothing otherwise; she will not be willing to either bet or accept bets if P is included between P_* e P^* (Fig. 1).

Figure 1



In these circumstances, the decision-maker's beliefs will be represented by the interval $[p_*(q), p^*(q)]$, where $p_*(q) = P_*/S$ and $p^*(q) = -P^*/-S$. Bewley's multiple prior approach has an interesting consequence for the completeness assumption.[25] When the decision-maker's preferences satisfy the completeness assumption, she can always compare any two state-contingent consumption bundles and decide which one she prefers on the basis of their respective expected utilities. If the decision-maker's beliefs are here represented by a multiplicity of beliefs, the decision-maker has to compute many expected utilities for each consumption bundle. A comparison between alternatives will be then carried out 'one probability distribution at a time' and one bundle will be preferred to another only if it is preferred under every probability distribution considered by the decision-maker. This makes it possible to characterise aversion to uncertainty: the greater the set of subjective probability distributions, the higher the set of consumption bundles that cannot be ranked uniformly, and the more reluctant the decision-maker to take action. Uncertainty aversion thus explains why people might respond to uncertainty by refraining from making a choice and taking an action (Rigotti and Shannon 2005).

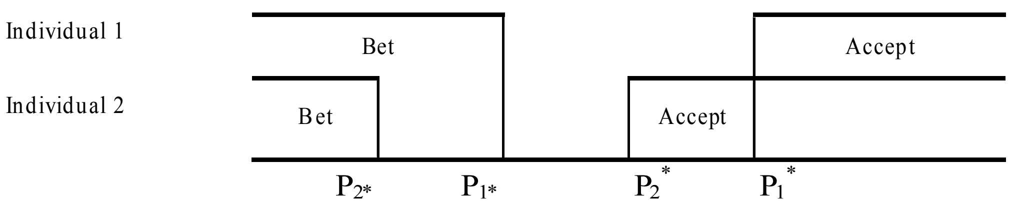
It is not difficult to understand the consequences of the multiple prior approach for the functioning of the insurance market. When individuals are endorsed with an interval of probabilities the multiple priors approach explains why they can have different opinions based on the same information and yet not want to exchange bets. At an intuitive level, the problem can be analysed by examining again the decision-maker's behaviour in betting situations. Let

25 The completeness assumption requires that for every pair of options x and y, the decision-maker either prefers x to y, prefers y to x, or is indifferent between them.

us suppose that two individuals, 1 and 2, have to decide whether to exchange a bet that pays S if the event E occurs and 0 otherwise.[26] The probability of E is unknown and the bet is not part of the two individuals' initial choice set. Let us imagine that the individual 1 regards the value of the lottery as included in the interval $[P_{1*}, P_1^*]$, where $P_1^* > P_{1*}$. As shown in the previous section, in these circumstances, she will be willing to pay each amount $P < P_{1*}$, and to accept bets for each amount $P > P_1^*$; she will instead shun bets for each amount P included in the interval $[P_{1*}, P_1^*]$. The individuals' degrees of belief will be thus represented by the interval included between $p_{1*} = P_{1*} / S$ e $p_1^* = -P_1^* / -S$. Similarly, let us suppose that the individual 2 regards the value of the lottery as included in the interval $[P_{2*}, P_2^*]$, where $P_2^* > P_{2*}$.. She will be thus willing to pay each amount $P < P_{2*}$, and to accept bets for each amount $P > P_2^*$; she will instead shun bets for each amount P included in the interval $[P_{2*}, P_2^*]$. The individuals' degrees of belief will be thus represented by the interval included between $p_{2*} = P_{2*} / S$ and $p_2^* = -P_2^* / -S$.

The possibility of organizing a bet which is convenient for both individuals depends on the values the decision-makers attach to P_* and P^*, and thus to their interval priors. In particular, individuals will exchange a bet if and only if the minimum probability attached to the event E by one decision-maker is greater than the maximum probability attached to the same event by the other decision maker (Bewley 1986, 1989). Whenever the price intervals $[P_*, P^*]$ overlap, the bet will not be exchanged (Fig. 2).

Figure 2



On the multiple prior approach, two rational decision-makers might express widely different beliefs about some outcome and yet not be willing to exchange a bet on that outcome. And since taking out insurance is equivalent to taking a bet on the subjective view, this kind of

26 For the sake of simplicity, we ignore the decision-makers' risk aversion.

situation is analogous to the absence of markets for the insurance of uncertain events (Bewley 1989).

6. Empirical evidence on insurance under ambiguity

As the material summarised above suggests, de Finetti's discussion of the key elements underpinning Knight's and Keynes's analysis of uncertainty makes a theoretical case for uncertainty within the subjectivist approach and provides solid theoretical ground for understanding the failure of insurance market to cover highly uncertain events. Obviously, that de Finetti's interpretations of Knightian and Keynesian uncertainty may explain an economic phenomenon at a theoretical level does not make it true in a descriptive sense. Only careful empirical work can establish whether the implications of Knightian and Keynesian uncertainty that we have been spelling out actually affect the practice of the insurance industry.

In the last twenty years or so, several empirical contributions have shown that this is indeed the case. A number of studies (Hogarth and Kunreuther 1985, 1989, 1995; Kunreuther and Hogarth 1992; Kunreuther et al. 1993; Kunreuther et al. 1995; Cabantous 2007) have investigated the decision process of actuaries, underwriters and reinsurers in setting premia for uncertain events and have shown that all three of these groups exhibit a large degree of uncertainty aversion. Accordingly, the empirical analysis of an increasing number of scholars has tried to explain the failure of the insurance market to cover highly uncertain contingencies by reviving a distinction between risk and uncertainty under the headings of ambiguity.[27]

In particular, following Smithson (1999), Cabantous (2007) surveyed insurance professionals and found that ambiguity aversion is pervasive in this population as they tend to raise premia above the levels they would charge when numerically definite probabilities are known. She also finds that sources of ambiguity (conflict of expert opinion or imprecision)

27 As in most of the recent developments in decision theory, following Ellsberg (1961), the notion of ambiguity is used to identify situations in which the individuals' information concerning the likelihood of events is perceived to be scanty or unreliable. Einhorn and Hogarth (1986, p. 227) define ambiguity as 'uncertainty about uncertainties'. They claim that when assessing uncertainty in the real world the analogy with gambling devices can be misleading, as 'beliefs about uncertain events are typically loosely held and ill defined'. On this view, ambiguity can be high when 'evidence is unreliable and conflicting' (Einhorn and Hogarth 1986, p. 230). The notion of ambiguity used in the works we refer to in this section mostly overlaps with that of uncertainty discussed in the previous sections. However, this literature does not refer to de Finetti, nor to the notions of Knightian uncertainty and Keynesian uncertainty discussed in this paper.

matter. To be specific, Cabantous (2007) distinguishes between situations of disagreement among experts about the probability of the occurrence of a given event (ambiguous and conflicting situations) and situations in which experts agree that the probability of a given event belongs to a range of possible values (ambiguous and consensual situations). She analyses the effects on the insurance markets of the two situations by collecting data from 78 professional actuaries, all members of the French Institute of Actuaries. Each of these people received a questionnaire containing scenarios involving pollution and an earthquake, with three informational conditions provided in each case. For instance, in the case of pollution, the insurer is provided with the following information:

- Risk - she knows that environmental studies establish with precision that the probability of pollution damage in the firm is $p = 0.002$;
- Ambiguity and consensual – experts agree that the probability of pollution damage in the firm is within the interval [0.001, 0.003], and that the mean probability of the risk is $p = 0.002$;
- Ambiguity and conflicting – one group of experts estimates that the probability is equal to $p = 0.001$, while the other group estimates that the probability is $p = 0.003$.

The insurer has to indicate the premium she would charge to insure firms against the risk and she can reject the insurer demand by setting up a premium that is too high to find a demand in the market.

It is thus not difficult to see that the two ambiguous scenarios reflect the conception of uncertainty stemming from the work of Keynes and Knight, both of which we have assessed with reference to de Finetti. The first definition of uncertainty is represented by an 'ambiguous and consensual situation', and the second one by the 'ambiguous and conflicting situation'. Cabantous's results confirm that insurers react to both notions of uncertainty by setting up a higher premium than in the risk situation. Further, the required premium in the conflicting situation is greater than the required premium in the ambiguous and consensual situation, that is, professional actuaries are more averse to ambiguity coming from conflict (some people agree on an upper bound and some other people agree on a lower bound) than ambiguity coming from imprecision (everybody agrees on a range of probability).[28]

This empirical analysis is of major importance for our reconstruction as it shows that

28 The main explanation of this phenomenon comes from the underlying hypothesis that 'experts should converge' (Shanteau, 2001).

the two sources of uncertainty must be distinguished as they may have different implications for the insurance market.

7. Conclusions

The economics of insurance provides a clear case of interaction between philosophy and economics. Scholars interested in the philosophical foundations of probability such as Knight and Keynes have devoted great attention to the possibility of applying probabilistic reasoning to the insuring of single events. We have argued that despite the philosophical differences in the foundations of their respective theories of probabilities, both Knight and Keynes suggest that insurance markets cannot provide insurance against truly 'uncertain' events since in this case probabilities either do not exists or cannot be measured, and maintain that the practice of the Lloyd's of London to insure single events does not prove that sharp numerical probabilities can be assigned to all events.

However, Knight's and Keynes's philosophical positions on this subject have been mostly disregarded since the distinction between risk and uncertainty has become irrelevant in the eyes of economists accustomed to the Subjective Expected Utility (SEU) model associated with Savage (1954), and which incorporates the 'orthodox' subjective conception of probability stemming from the work of Bruno de Finetti (1937) himself. The standard model of risk exchange which, relying on the use of a subjective approach to probability stemming from the work of de Finetti, predicts that all individual uncertainties will be insured, and competition on the insurance market will lead to a Pareto-efficient allocation of risk in the economy.

By relying on some overlooked excerpts from de Finetti's vast contribution to economics and statistics published in Italian, we have argued that he showed an attitude towards the issue of uncertainty and its justification in the theory of probability that does not conform with the traditional representation of him as the champion of a strictly subjective approach, and opens up the possibility of considering de Finetti's position closer to a broader subjective perspective. We have shown that de Finetti actually made a theoretical case for uncertainty within the subjectivist approach by reinterpreting Knight's and Keynes's distinction between risk and uncertainty within a pure subjectivist framework. And we have argued that despite the failure of the economic literature to recognize these reinterpretations, there is increasing empirical evidence showing that both versions can help explain the difficulties of the insurance market in covering highly uncertain events.

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